Dynamical screening in hot systems away from (chemical) equilibrium *)

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Within the Closed Time Path Formalism of Thermal Field Theory we calculate the hard photon emission rate as well as the collisional energy-loss rate for a quark-gluon plasma away from chemical equilibrium. Mass singularities are shown to be dynamically screened within HTL-resummed perturbation theory also away from equilibrium. Additional (pinch) singularities are absent and well defined results are obtained.

§1. Introduction

At present, major efforts are being made to experimentally probe a new deconfinement state of strongly interacting matter (Quark-Gluon Plasma, QGP), and much related research is being done in order to establish a sufficient theoretical understanding of the systems under investigation here. As a central result of research done to date, the hard-thermal-loop (HTL) resummation scheme [1)-3)] allows to take collective effects into account within an improved perturbative approach. For systems in thermal equilibrium, based on HTL-effective perturbation theory, predictions for potentially observable emission rates can be obtained, with medium effects providing the required (dynamical) screening of singularities.

More recently, need for a better theoretical understanding of non-equilibrium effects has become apparent, since systems actually under investigation in a heavy-ion collision are likely to stay away from equilibrium for important periods. In this work, we attempt to study the role of collective effects in systems away from equilibrium and to extend the HTL-resummation prescription appropriately. We will study hard thermal photon production [4)-10)] as well as the collisional energy loss [11)-13)], which both are known to be sensitive to soft scale physics. The following is based on [14), 15)], where further details can be found.

§2. The approximation scheme - chemical non-equilibrium

The scenario of chemical non-equilibrium [16)-19)] has been argued to be a valid simplification of the actual, complicated pre-equilibrium dynamics in a heavy-ion collision. It is inspired from an analysis of the processes leading to equilibration eventually, from which the effect of elastic collisions is more pronounced than the effect of inelastic processes. Consequently, while local thermal equilibrium might be quickly established, chemical equilibrium among the different particle species and their respective number-densities will be delayed. In particular the number density of quarks is expected to be low while gluons will reach their equilibrium density more quickly, as has been formulated in the hot-glue scenario [16)].

In this work we follow the analysis, e.g., of 17), parameterizing chemical non-equilibrium in terms of fugacity factors λ multiplying the distribution functions $\tilde{n}(X,p) = \lambda_q(X) n_F$ for quarks and $n(X,p) = \lambda_g(X) n_B$ for gluons [9), 20)]. Here n_F, n_B are the equilibrium Fermi- and Bose-distributions respectively, and the (space-) time variable X refers to the scale of chemi-

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cal equilibration. We do not attempt to analyze the process of equilibration but assume the evolution of $\lambda_{q,g}$ as input, e.g., from 17). As a function of these, we attempt to determine photon production and energy-loss rates. Based on the Closed-Time-Path formalism of Thermal Field Theory [21), 22)] and within the simplified scenario of chemical non-equilibrium, we consider only lowest (second) order initial correlations and the scale of equilibration can be argued to be $X \sim 1/g^4T$. Scales fast with respect to X, in particular both hard and soft scales, can than be transformed to momentum space (Wigner transformation). To lowest order of an expansion in the gradients of equilibration [22) - 25)], the resulting perturbation theory is very similar to the equilibrium formalism with bare propagators for scalars and fermions respectively

$$iD_{21} = 2\pi\varepsilon(p_0)\delta(p^2)(1+n(X,p_0)), \quad iS_{21}(X,p) = \not p\, 2\pi\delta(p^2)\varepsilon(p_0)(1-\tilde n(X,p_0)),$$

now depending on the modified distribution functions. Physical rates calculated in this way are expected to depend on λ_q, λ_g via $\tilde n(X,p)$ and $n(X,p)$. However, when investigating quantities sensitive to soft-scale physics it will be necessary to extend the HTL-resummation scheme to the present situation away from equilibrium. Doing so the required partial resummation of selfenergy corrections is known to entail contributions not present in the equilibrium analysis [26), 27)], which should be taken into account.

§3. Soft fermion exchange in hard real photon production

To lowest order in perturbation theory, real photon production from the QGP arises from Compton and annihilation processes. The respective production rates can be calculated from the absorptive part of the 2-loop selfenergy diagram shown in Fig. 1a. At fixed lowest order, a divergent result is known to arise from a mass-singularity induced by the exchange of a massless quark in the process. However, in case of an equilibrium system, with the dispersion of soft quarks being modified within the HTL-framework, the singularity is found to be screened by Landau-damping effects and a well defined result is obtained [4), 5)].

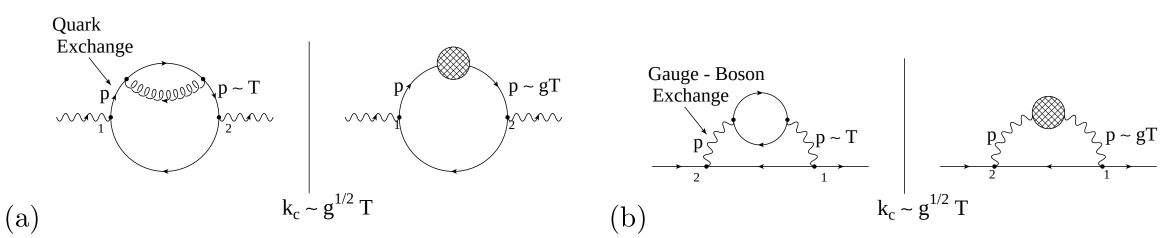

Fig. 1. Diagrams contributing to hard photon production (a) and collisional energy-loss rates (b). The lefthand side of each display shows fixed order of perturbation theory for hard exchanged momentum, while the right-hand side indicates HTL-resummation for soft exchanged momentum. The parameter k_c separating hard and soft regions drops out in the sum of hard and soft partial rates. [28)]

Away from equilibrium, the resummed propagator for soft quark exchange can be written

$$\begin{aligned} S^\star_{12}(p) = &-\tilde n(X,p_0)\Big(\frac{1}{\not p - \Sigma_R + i\varepsilon p_0} - \frac{1}{\not p - \Sigma^\star_R - i\varepsilon p_0}\Big) \\ &-\frac{1}{\not p - \Sigma_R + i\varepsilon p_0}\left[(1-\tilde n(X,p_0))\Sigma_{12} + \tilde n(X,p_0)\Sigma_{21}\right]\frac{1}{\not p - \Sigma^\star_R - i\varepsilon p_0}. \end{aligned} \tag{3·1}$$

Here, the first term on the right hand side has a structure parallel to the equilibrium expression, while the second term vanishes if detailed balance holds and contributes only away from equilibrium. There are two points to be made. First, non-equilibrium modifications to the dispersion,

as defined by the retarded propagator in the first term of Eq. (3·1), amount to a modification of the soft-scale parameter m_q only:

$$\tilde{m}_q^2 = \frac{g^2}{2\pi^2} C_F \int_0^\infty E dE (n(X,E) + \tilde{n}(X,E)) = \frac{g^2 T^2}{12} C_F \left(\lambda_g + \frac{\lambda_q}{2} \right). \qquad (3\cdot2)$$

The reason for this is that HTL-contributions to the self-energy, just as in the equilibrium case, come proportional to the integral displayed in the definition of $\tilde{m}_q$, Eq. (3·2). Next, in order to discuss the second term on the right-hand side of Eq. (3·1) we propose to rewrite it according to

$$(1 - \tilde{n}(X,p_0))\Sigma_{12} + \tilde{n}(X,p_0)\Sigma_{21} = (1 - 2\tilde{n}(X,p_0))\Sigma^- + \Sigma^+. \qquad (3\cdot3)$$

Here, $\Sigma^- = \frac{1}{2}(\Sigma_{12} - \Sigma_{21}) = iIm\Sigma_R$ and $\Sigma^+ = \frac{1}{2}(\Sigma_{12} + \Sigma_{21})$ have been introduced. In the fermionic case, the contribution from Σ^+ is found to be small compared to the HTL-contribution in Σ^-. Explicitly we find

$$\Sigma^+(X,p) \sim p_0 R\, \Sigma^-(X,p) \ll \Sigma^-, \quad R = \frac{\int_0^\infty dk\ k\ \frac{\partial}{\partial k}\tilde{n}(k)(1+2n(k))}{\int_0^\infty dk\ k\ [n(k) + \tilde{n}(k)]} \sim \frac{1}{T}, \qquad (3\cdot4)$$

so that for $p_0 \sim gT$, $\Sigma^+ \sim g^3T^2$ results. Consequently, within the HTL-scheme, the contribution from Σ^+ to the HTL-resummed propagator can be neglected. Doing so and using furthermore

$$2\Sigma^- = \Sigma_R - \Sigma_A = {S_A^\star}^{-1} - {S_R^\star}^{-1}, \qquad (3\cdot5)$$

the effective propagator can be simplified into

$$\begin{aligned} S_{12}^\star|_{HTL} &\simeq -\tilde{n}(X,p_0)\,(S_R^\star - S_A^\star) - (\frac{1}{2} - \tilde{n}(X,p_0)) S_R^\star [2\Sigma^-] S_A^\star \\ &= -\frac{1}{2}\,(S_R^\star - S_A^\star)\,. \end{aligned} \qquad (3\cdot6)$$

In this form, $S_{12}^\star$ has the same structure as in equilibrium so that the remaining phase space integrations can be performed as explained in the respective literature [4), 5)] taking only the modification of the dispersion, Eq. (3·2), into account. The hard photon production rate from a system away from equilibrium is obtained as

$$\begin{aligned} E_\gamma \frac{dR}{d^3q} = e_q^2 \frac{\alpha\alpha_s}{2\pi^2} \lambda_q T^2 e^{-E_\gamma/T} \Bigg[&\frac{2}{3}(\lambda_g + \frac{\lambda_q}{2}) \ln\left(\frac{2E_\gamma T}{\tilde{m}_q^2(\lambda_q,\lambda_g)} \right) \\ &+ \frac{4}{\pi^2} C(E_\gamma, T, \lambda_q, \lambda_g) \Bigg], \end{aligned} \qquad (3\cdot7)$$

where the finite contribution $C(E_\gamma, T, \lambda_q, \lambda_g)$ has been given explicitly in 14). The important observation in Eq. (3·7) is that the logarithmic divergence turns out to be screened on the scale of the parameter $\tilde{m}_q(\lambda_q, \lambda_g)$. The result, Eq. (3·7), is independent of the parameter k_c upon summation of soft and hard partial rates. The noteworthy point here is that no additional (pinch) singularities [26), 27)] arise from the second term in the propagator, Eq. (3·1), when calculating to one-loop order. This is because phase space for real photon production is restricted to space-like momentum exchange, and the pinch-singular region on the mass-shell does not contribute in this case. The propagator for the line of hard quark exchange in Fig. 1a, as given by the one-loop approximation to Eq. (3·1), can than be written as

$$\delta S_{12}(p)|_{p^2 \leq -k_c^2} \triangleq -\frac{1}{(p^2)^2} \not{p}\, \Sigma_{12} \not{p}\,, \qquad (3\cdot8)$$

which, similar to Eq. (3·6), does not depend explicitly on the distribution $\tilde{n}(X,p)$. The remaining mass-singularity turns out to be dynamically screened in Eq. (3·7).

§4. Exchange of a soft boson in the collisional energy loss

Next we consider the collisional energy-loss for a charged particle going through a QED medium assumed to be away from equilibrium. We consider a heavy external probe with mass $M \gg T$ [12), 13)]. To lowest order the energy-loss arises from elastic collisions with particles in the medium and can be obtained from the (21)-component of the selfenergy of the projectile as shown in Fig. 1b:

$$\begin{aligned} -\frac{dE}{dx} &= -\frac{1}{4E} Tr\left[(\not{q} + M) i\Sigma'_{21}(X, q)\right] \\ &= \frac{e^2}{4E} \int \frac{d^3\vec{p}}{(2\pi)^4} \int \frac{dp_0 p_0}{v} Tr[(\not{q} + M)\gamma_\mu i S_{12}^{T=0}(p-q)\gamma_\nu] i D_{21}^{\mu\nu}(X, p). \end{aligned} \quad (4\cdot1)$$

Here, an additional power of p_0, the energy exchanged, is present in the integrand, which, for an equilibrium systems, makes the energy-loss finite after resummation of HTLs, in contrast with the fermion damping rate.

In this case, in contrast with the exchange of a soft fermion as considered in the previous section, we investigate the exchange of a soft (gauge) boson. The longitudinal and transverse components of the corresponding resummed propagator can be expressed as [29)]

$$\begin{aligned} D_{21}^{\star L/T} &= (1 + n(X, p_0))(D_R^{\star L/T} - D_A^{\star L/T}) \\ &\quad + D_R^{\star L/T}\left[n(X, p_0)\Pi_{21}^{L/T} - (1 + n(X, p_0))\Pi_{12}^{L/T}\right] D_A^{\star L/T}. \end{aligned} \quad (4\cdot2)$$

The propagator D_{21}, Eq. (4·2), can be analyzed along the same lines as the fermion propagator, Eq. (3·1). Namely, with regard to the dispersion the only non-equilibrium modification concerns the plasma frequency, which is redefined according to

$$m_\gamma^2 = \frac{e^2 T^2}{9} \quad \rightarrow \quad \tilde{m}_\gamma^2 = \frac{4e^2}{3\pi^2} \int_0^\infty dk k \tilde{n}(X, k) = \lambda_f m_\gamma^2. \quad (4\cdot3)$$

However, when investigating the second term in D_{21}, Eq. (4·2), the contribution of $\Pi^+ = \frac{1}{2}(\Pi_{12} + \Pi_{21})$ turns out to be larger than that of $\Pi^- = \frac{1}{2}(\Pi_{12} - \Pi_{21}) = i Im \Pi_R$ [29)]

$$-\Pi_{L/T}^+ = \frac{1}{2p_0}\, R\, \Pi_{L/T}^-, \quad R = \frac{\int_0^\infty dk k^2 \tilde{n}(k)(1 - \tilde{n}(k))}{\int_0^\infty dk k \tilde{n}(k)} \sim T, \quad (4\cdot4)$$

and Π^+ may not be neglected in general. However, in the energy-loss the contribution of Π^+ integrates to zero given its odd symmetry under $p_0 \rightarrow -p_0$:

$$\int_{-vp}^{vp} dp_0 p_0 \Pi^+(X, p_0) = 0. \quad (4\cdot5)$$

Therefore, there is no contribution from Π^+ to dE/dx, and the propagator, Eq. (4·2), effectively simplifies to

$$D_{21}^{\star L/T}\Big|_{dE/dx} \stackrel{\wedge}{=} \frac{1}{2}(D_R^{\star L/T} - D_A^{\star L/T}) \qquad \text{in} \quad dE/dx. \quad (4\cdot6)$$

Again this form is similar to the equilibrium expression, and the phase-space integration may be performed as in 12). As for real photon production, pinch singularities are absent since only space-like momentum exchange is involved, and the remaining mass singularity is dynamically screened, providing a well defined result.

When generalizing to the case of a heavy quark propagating through a QGP [13] one obtains for $E \ll M^2/T$,

$$-\frac{dE}{dx} = \frac{g^2 \tilde{m}_g^2}{2\pi v}\left(1 - \frac{1-v^2}{2v}\ln\frac{1+v}{1-v}\right)\left[\ln\frac{ET}{3M\tilde{m}_g} - \frac{\ln 2}{(1+\lambda_q N_f/6\lambda_g)} + A(v)\right], \quad (4{\cdot}7)$$

and for $E \gg M^2/T$,

$$-\frac{dE}{dx} = \frac{g^2 \tilde{m}_g^2}{2\pi}\ln\left[0.920\frac{\sqrt{ET}}{\tilde{m}_g}\,2^{\lambda_q N_f/(12\lambda_g + 2\lambda_q N_f)}\right]. \quad (4{\cdot}8)$$

Here, g is the QCD coupling constant, $\tilde{m}_g = g^2 T^2(\lambda_g + \lambda_q N_f/6)/3$ is the appropriate generalization of $\tilde{m}_f$, Eq. (4·3), now depending on both the quark and the gluon fugacities and finally $A(v)$ a remaining finite contribution [13]. In Fig. 2 the result is summarized for a charm quark, where for $\lambda_g = 1, \lambda_q = 0$ the loss is due to elastic gluon-heavy quark scattering mediated by gluon exchange.

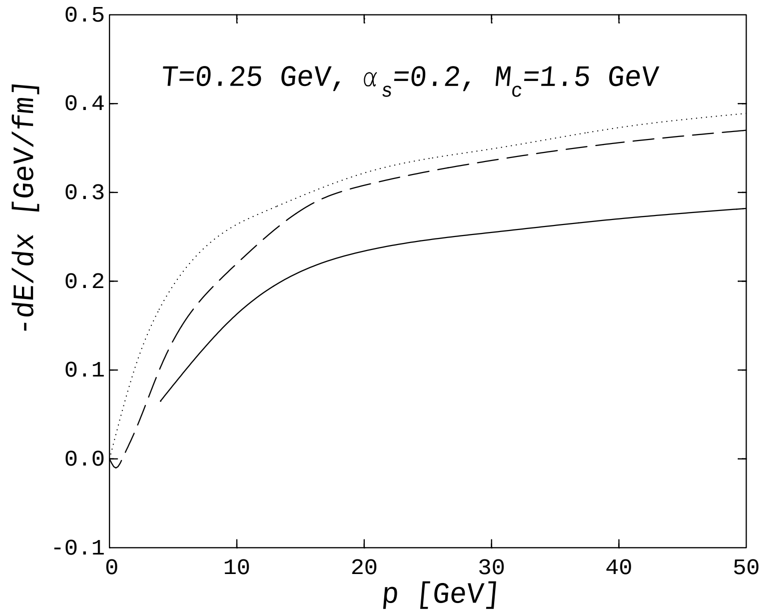

Fig. 2. Collisional energy loss of a charm quark as a function of its momentum. The quark propagates through an out-of-chemical equilibrium plasma with fugacities $\lambda_g = 1, \lambda_q = 0$ (solid curve). The dashed curve is the equilibrium result of [13], the dotted curve shows the original prediction by Bjorken [11] .

§5. Conclusion

In this work we investigated interaction processes occurring in a plasma away from chemical equilibrium, namely the production of hard real photons and the collisional energy-loss. The important observation made is that dynamical screening is effective also away from equilibrium, and finite predictions, (3·7) and (4·7), can be given for the respective rates. Both rates depend on the fugacities λ_q, λ_g, so that specific predictions with respect to actual heavy-ion experiments necessarily require further assumptions on the expansion and equilibration dynamics.

Finally, it is necessary to point out that the absence of pinch singularities, in the way observed here, may not be valid in general. Namely in the production of leptons (virtual photons) contributions from time-like momentum exchange arise and a more elaborate approach might have to be developed [30)-32)].

§6. Acknowledgments

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